

B-spline Methods in Fluid Dynamics

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Abstract

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B-spline functions are bases for piecewise polynomials that possess attractive properties for complex flow simulations : they have compact support, provide a straightforward handling of boundary conditions and grid nonuniformities, and yield numerical schemes with high resolving power, where the order of accuracy is a mere input parameter. This paper reviews the progress made on the development and application of B-spline numerical methods to computational fluid dynamics problems. Basic B-spline approximation properties is investigated, and their relationship with conventional numerical methods is reviewed. Some fundamental developments towards efficient complex geometry spline methods are covered, such as local interpolation methods, fast solution algorithms on cartesian grid, nonconformal block-structured discretization, formulation of spline bases of higher continuity over triangulation, and treatment of pressure oscillations in Navier-Stokes equations. Application of some of these techniques to the computation of viscous incompressible flows is presented.

Key Words: B-splines; Galerkin & collocation methods; Mass matrix; Fast solvers; Block-structured grids; Triangulated splines; Local refinement; Incompressible Navier-Stokes equations; Turbulent flows.

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